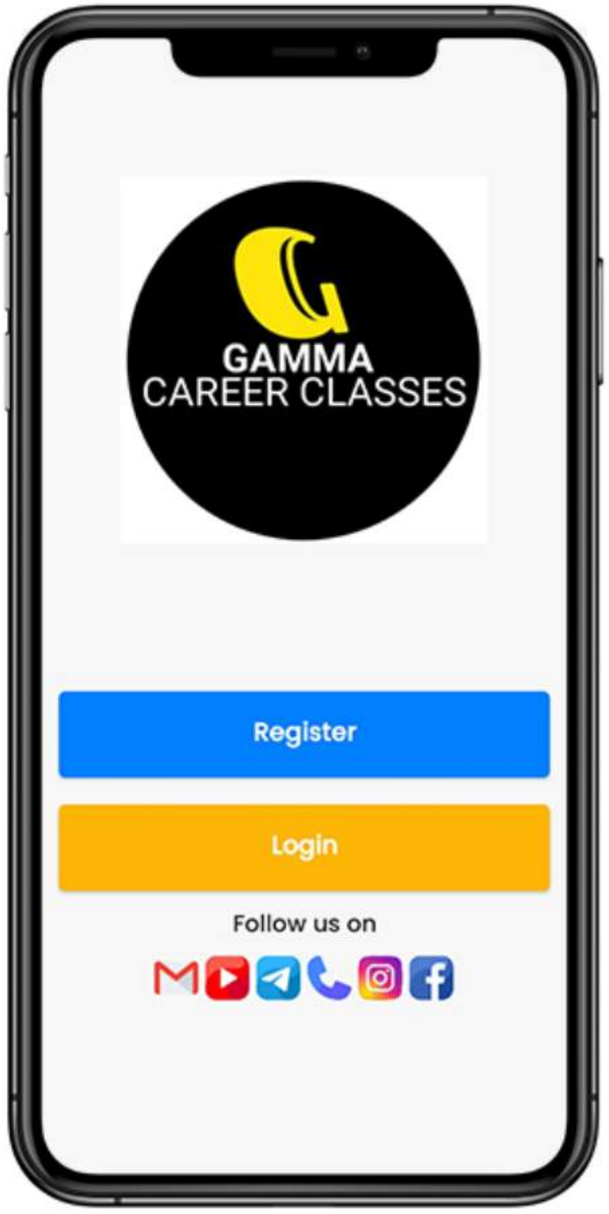
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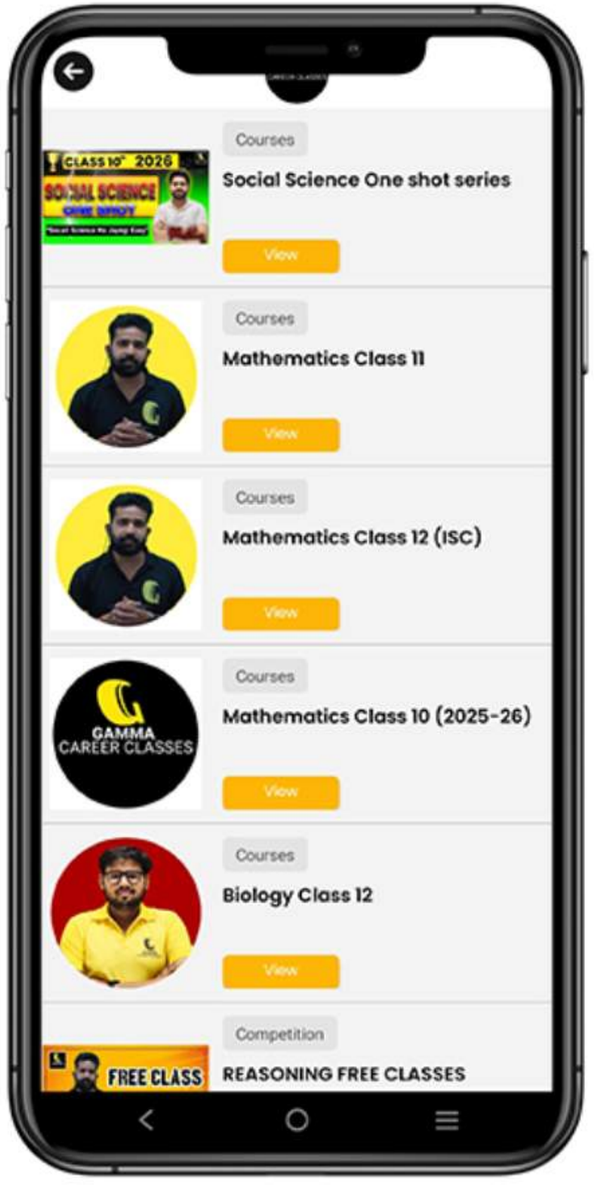
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Three Dimensional Geometry

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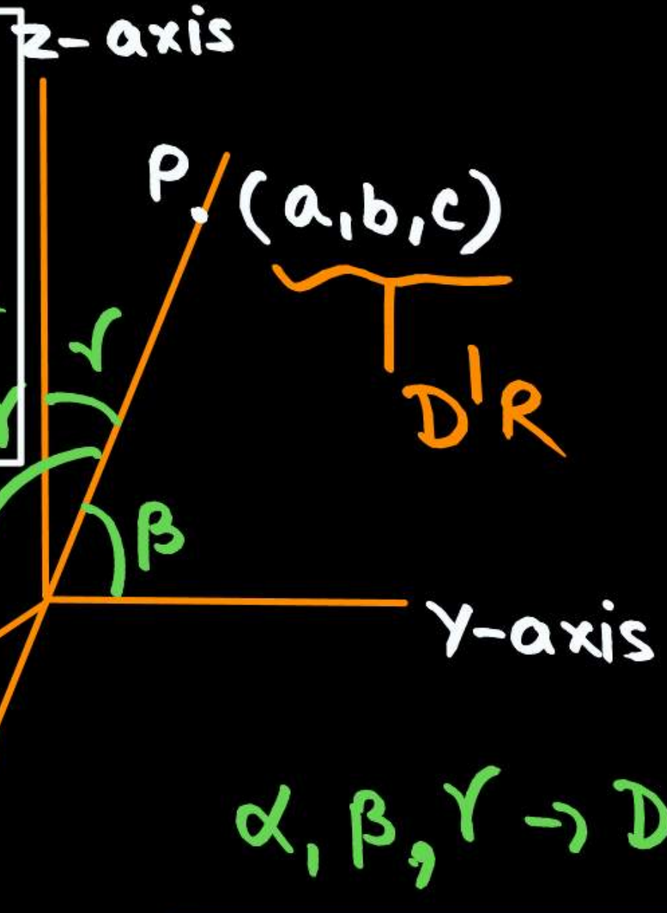
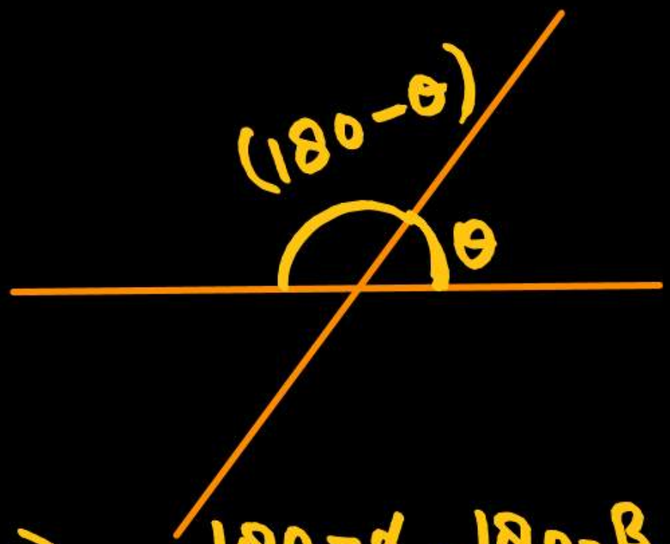
Direction cosine and Ratio of line

Direction Ratio
(a, b, c)

Direction Cosine
$\cos\alpha, \cos\beta, \cos\gamma$



$$l^2 + m^2 + n^2 = 1$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



$\alpha, \beta, \gamma \rightarrow$ Directed angle $\rightarrow$ $180-\alpha, 180-\beta, 180-\gamma$

$\Rightarrow \cos(180-\alpha), \cos(180-\beta), \cos(180-\gamma)$

$\Rightarrow -\cos\alpha, -\cos\beta, -\cos\gamma$

$\cos\alpha, \cos\beta, \cos\gamma$ — D.C.
$l \quad m \quad n$

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If a line makes angle $90^0, 135^0$ *and* 45^0 with the position direction of X,Y,Z axes then Find the direction Cosine.

$\cos\alpha, \cos\beta, \cos\gamma$

x-axes = 90°

Y-axes = 135°

Z-axes = 45°

$l = \cos\alpha = \cos 90^\circ = 0$

$m = \cos\beta = \cos 135^\circ = \frac{-1}{\sqrt{2}}$

$n = \cos\gamma = \cos 45 = \frac{1}{\sqrt{2}}$

D'C → $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

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Direction Cosine and Direction Ratio always proportional

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c}=k$$

$$l^2+m^2+n^2=1$$

$$l=ak \quad m=bk \quad n=ck$$

$$(ak)^2+(bk)^2+(ck)^2=1$$

$$a^2k^2+b^2k^2+c^2k^2=1$$

$$k^2[a^2+b^2+c^2]=1$$

$$k^2=\frac{1}{a^2+b^2+c^2}$$

$$k=\frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m=\frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

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If a line has direction ratios 2,-1,-2 determine its direction cosines.

$a = 2 \quad b = -1 \quad c = -2$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-1}{3}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{-2}{3}$$

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Find the direction cosines of x, y and z axis.

Imp.

For x-axis

$\alpha = 0^\circ \quad \beta = 90^\circ \quad \gamma = 90^\circ$

$\cos\alpha, \cos\beta, \cos\gamma$

$\cos 0^\circ, \cos 90^\circ, \cos 90^\circ$

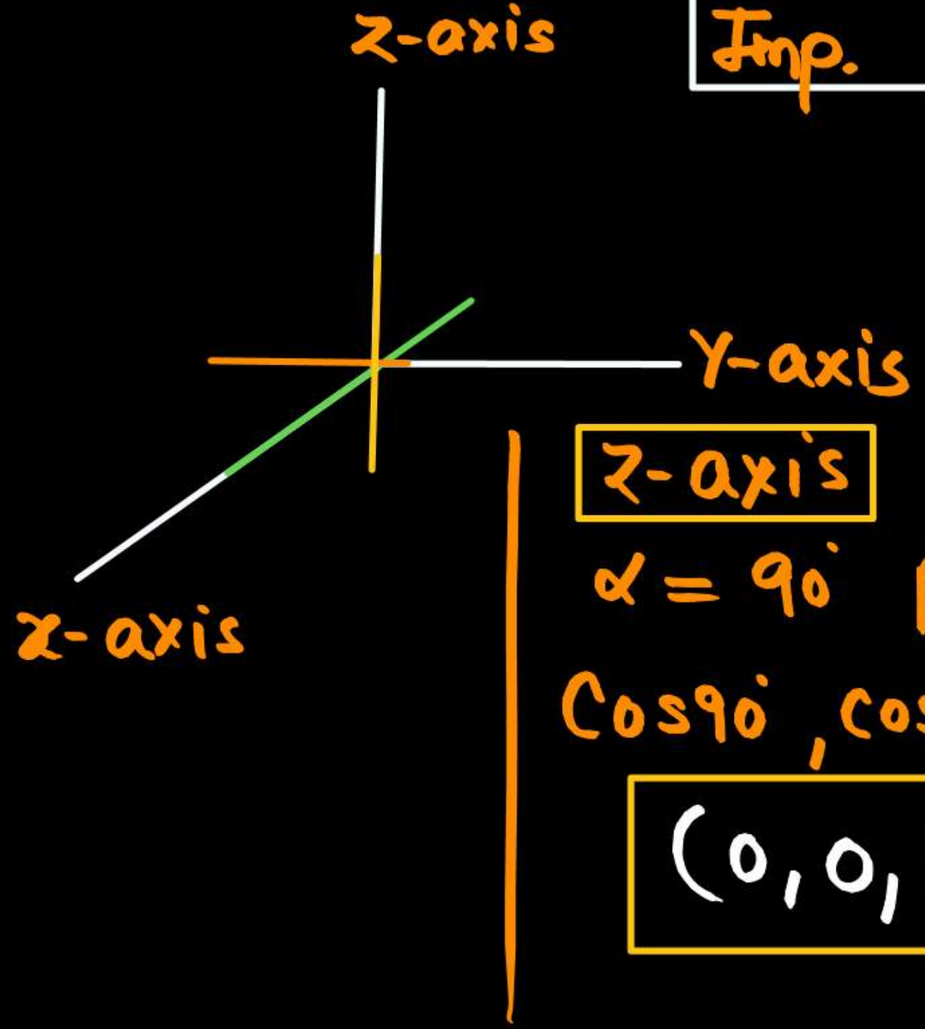
$(1, 0, 0)$

Y-axis

$\alpha = 90^\circ \quad \beta = 0^\circ \quad \gamma = 90^\circ$

$\cos 90^\circ, \cos 0^\circ, \cos 90^\circ$

$(0, 1, 0)$



Z-axis

$\alpha = 90^\circ \quad \beta = 90^\circ \quad \gamma = 0$

$\cos 90^\circ, \cos 90^\circ, \cos 0^\circ$

$(0, 0, 1)$

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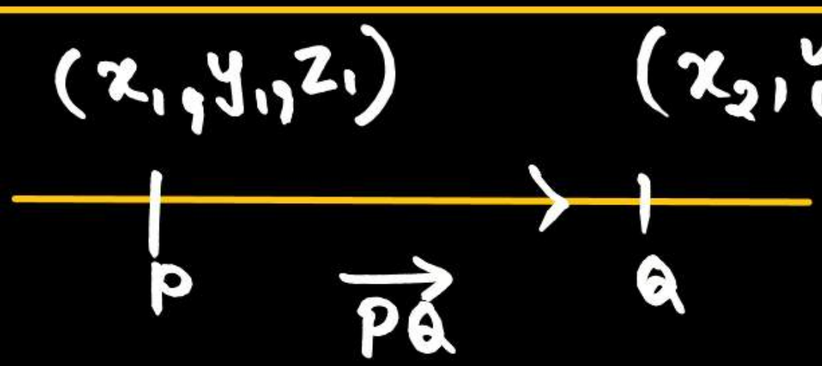
Direction cosine of A line Passing through two Points

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) -$

$(x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$



$\vec{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{OQ} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{PQ} = \underbrace{(x_2 - x_1)}_{a}\hat{i} + \underbrace{(y_2 - y_1)}_{b}\hat{j} + \underbrace{(z_2 - z_1)}_{c}\hat{k}$

D'R $(\underbrace{x_2 - x_1}_{a}), (\underbrace{y_2 - y_1}_{b}), (\underbrace{z_2 - z_1}_{c})$

D'C $\rightarrow \dfrac{(x_2 - x_1)}{\sqrt{a^2+b^2+c^2}}, \dfrac{y_2 - y_1}{\sqrt{a^2+b^2+c^2}}, \dfrac{z_2 - z_1}{\sqrt{a^2+b^2+c^2}}$

$l = \dfrac{a}{\sqrt{a^2+b^2+c^2}}$

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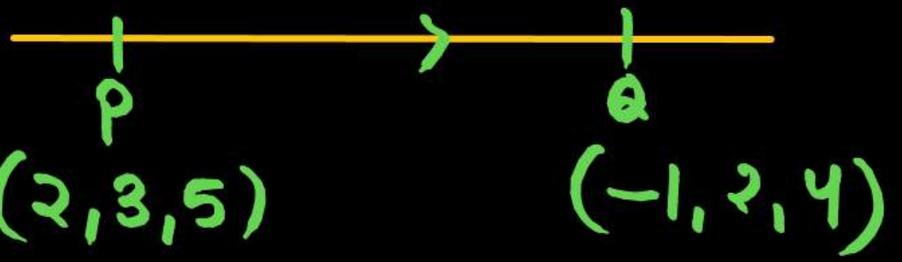
Find the direction Cosines of the line passing through the points P(2,3,5) and Q(-1,2,4)



$\overrightarrow{PQ} = (-1-2)\hat{i} + (2-3)\hat{j} + (4-5)\hat{k}$

$\overrightarrow{PQ} = -3\hat{i} - \hat{j} - \hat{k}$

D'R = $-3, -1, -1$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{-3}{\sqrt{(-3)^2+(-1)^2+(-1)^2}} = \frac{-3}{\sqrt{9+1+1}} = \frac{-3}{\sqrt{11}}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-1}{\sqrt{11}}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{-1}{\sqrt{11}}$$

$\frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}$ Ans.

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Show that the points A(2, 3,-4) B(1,-2,3) c(3,8,-11) are collinear

D'R of AB

$= (1-2), (-2-3), (3+4)$

D'R of AB = -1 , -5, +7

D'R of BC

$(3-1), (8+2), (-11-3)$

$2, 10, -14$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14}$$

$$-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$$

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A Line makes angle $60^0, 45^0, 60^0$ with x, y, z axes what is the value of n

$$l \quad m \quad n = \cos\gamma$$

$$n = \cos 60^\circ = \frac{1}{2}$$

$$\boxed{n = \frac{1}{2}}$$

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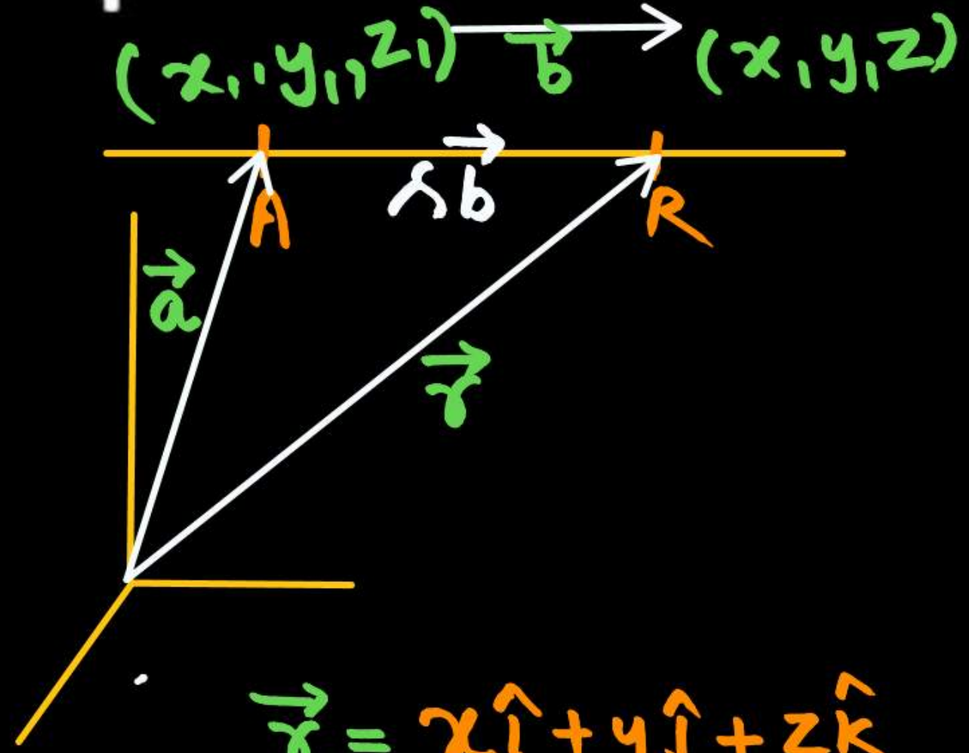
Equation of a line in space



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\vec{AR} = \lambda\vec{b}$$

$$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$$ — Vector form.

$$x\hat{i} + y\hat{j} + z\hat{k} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$$

$$x\hat{i} + y\hat{j} + z\hat{k} = (x_1 + a\lambda)\hat{i} + (y_1 + b\lambda)\hat{j} + (z_1 + \lambda c)\hat{k}$$

$$x = x_1 + a\lambda$$

$$x - x_1 = a\lambda$$

$$\boxed{\frac{x - x_1}{a} = \lambda}$$

$$y = y_1 + b\lambda$$

$$y - y_1 = b\lambda$$

$$\boxed{\frac{y - y_1}{b} = \lambda}$$

$$z = z_1 + c\lambda$$

$$z - z_1 = c\lambda$$

$$z - z_1 = c\lambda$$

$$\boxed{\frac{z - z_1}{c} = \lambda}$$

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$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Cartiesion Form

$x_1, y_1, z_1 \rightarrow$ passing point

$a, b, c \rightarrow D^1R$

$a, b, c \rightarrow D^1R$ of $\parallel$ vector.

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A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of $\hat{i} + \hat{j} - 2\hat{k}$. Find the Equations of the line in vector and in cartesian form

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

$\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ $(2, -1, 4)$

$\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ → D'R of ll Vector $(1, 1, -2)$

$$\vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$$

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$$

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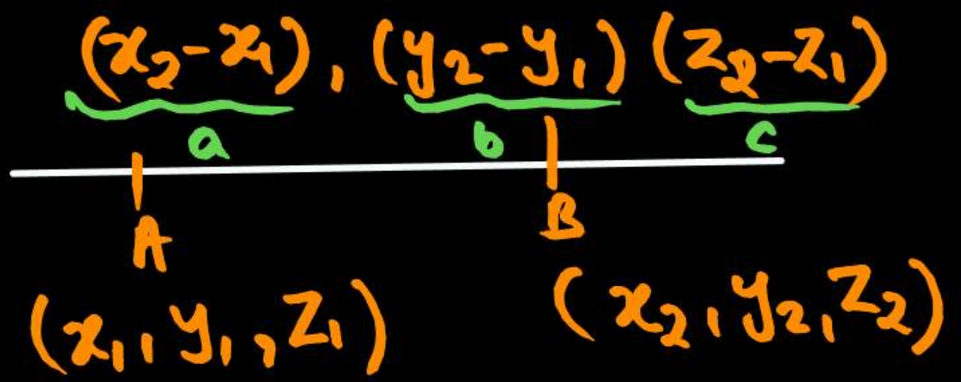
Equation of a line passing through two given point

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$



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Find the cartesian equation of the line which passes through the point (-2,4,-5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

D'R (3, 5, 6)

passing point (-2, 4, -5)

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

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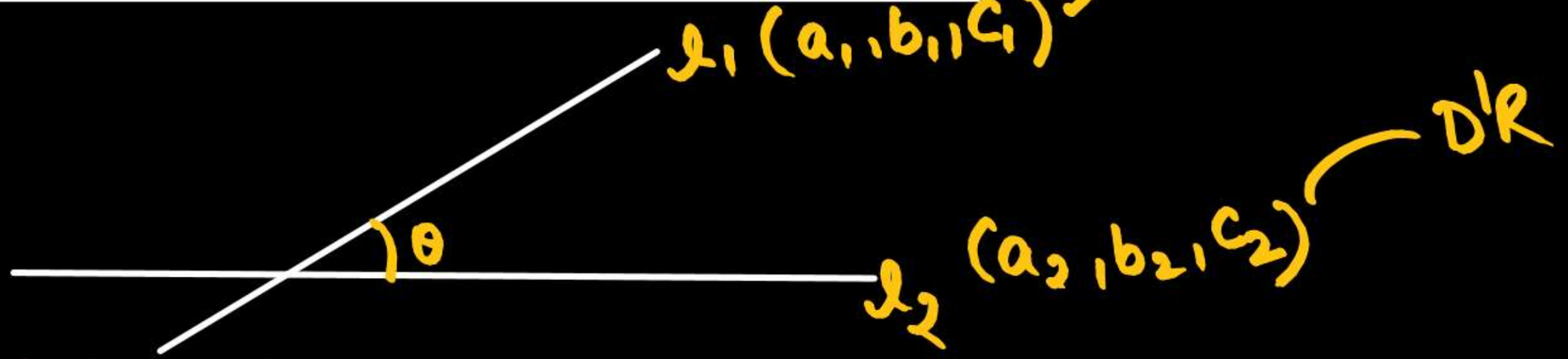
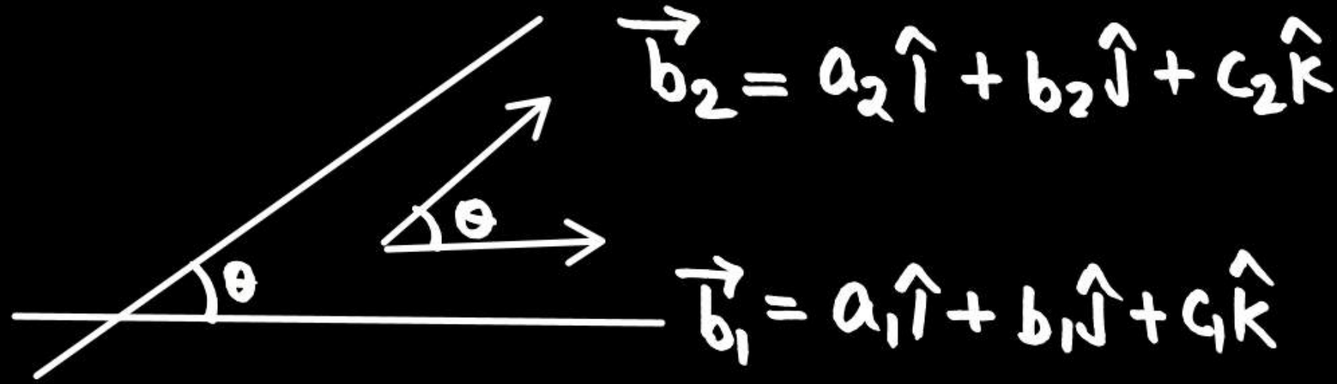
Angle between to line



$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\vec{b_1}\cdot\vec{b_2} = |\vec{b_1}||\vec{b_2}|\cos\theta$$

$$\cos\theta = \frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$$



$$\vec{b_2} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\vec{b_1} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

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Case → If line ⊥

$$\vec{b_1} \cdot \vec{b_2} = 0$$

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

Case-2 If line are parallel.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k$$

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If the line $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-2}{2} = \frac{z-6}{-5}$ are perpendicular. Find the value of λ.

$a_1 = -3 \quad b_1 = 2\lambda \quad c_1 = 2 \qquad a_2 = 3\lambda \quad b_2 = 2 \quad c_2 = -5$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$-3 \times 3\lambda + 2\lambda \times 2 + 2 \times -5 = 0$$

$$-9\lambda + 4\lambda - 10 = 0$$

$$-5\lambda - 10 = 0$$

$$-5\lambda = 10$$

$$\lambda = \frac{-10}{5} = -2$$

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Find the angle $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

$$\frac{x-(-3)}{3} = \frac{y-1}{5} = \frac{z-(-3)}{4} \qquad \frac{x-(-1)}{1} = \frac{y-4}{1} = \frac{z-5}{2}$$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{3\times1 + 5\times1 + 4\times2}{\sqrt{(3)^2+(5)^2+(4)^2}\sqrt{(1)^2+(1)^2+(2)^2}}$$

$$\cos\theta = \frac{3+5+8}{\sqrt{9+25+16}\sqrt{1+1+4}} = \frac{16}{\sqrt{50}\sqrt{6}}$$

$$\theta = \cos^{-1}\left[\frac{16}{\sqrt{50}\sqrt{6}}\right]$$

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Distance between two skew line

$$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$$
$$\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$$

$$\mathbf{d}=\left|\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$$

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Shortest distance between two line

$$\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$$
$$\vec{r} = \overrightarrow{a_2} + \lambda\vec{b}$$

$$d = \left| \frac{\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\vec{b}|} \right|$$

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Thank You!

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